"GAUSSIAN BEAM PROPAGATION VERSUS HUYGENS METHOD IN NON-PARAXIAL CONDITIONS"

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Abstract

In this paper, we present a study about the difference between Gaussian beam propagation and Huygens method to obtain the radiation pattern in the case of non-paraxial conditions.

We fix a way to determine the error in the Gaussian beam propagation, in this way, we could know if the approximation is good enough.

The computational results show that it is not possible to use the Gaussian propagation equation when we are working with Gaussian beams in non-paraxial conditions.

Introduction

There are many applications, as for instance systems using a quasi-optical transmission line between the source and the load (high power heating, material processing and ceramic sintering), in which free space eignmodes must be used [1], [2]. These ones are the preferred choice when high efficiency, maximum matching and low losses on the mirror system are required.

If the wave equation in free space is solved using paraxial condition, the gaussian beams are obtained as result. The exact expression can be found in [3].

Anyway, this solution is only an approximation to the real solution of the wave equation in free space.

By the way, the designers use the gaussian beams as free space solution obtaining easily the transmission lines features like position and size of the mirrors. Nevertheless, this will only be valid in the case of paraxial conditions.

In [4] and [5], an analysis between using the exact expression for the far field pattern of a gaussian amplitude distribution with constant phase, and the paraxial approximation under the form of the paraxial expressions for the gaussian modes is presented. There, a value of $kw_0>6$ is chosen to fix the paraxial condition. This value, shown in [5], is corresponds to an 2-Dimensional error of 3%.

In this paper, we present a paraxiality analysis using the Huygens method versus Gaussian propagation. We show graphically the difference between both methods and fix new error values. In this way, a correct design of the transmission line could be done.

Huygens Method versus Gaussian Propagation

The Huygens method consists on performing the direct integration of the electromagnetic field equations of the source over any surface, obtaining the real solution of the far field radiation pattern in three dimensions. In this case, the field distribution at the source is following a Gaussian profile.

The equations used to compute radiation pattern with this technique can be found in [6].

On the other hand, using the formulas shown in [3], we obtain directly the approximation of the gaussian beam radiation. In particular, the equations describing the gaussian expansion are the following :

$$\text{for} \quad z >> z_0 = \frac{k\varpi_0^2}{2} \qquad \varpi(z) = \varpi_0\sqrt{1+\left(\frac{2z}{k\varpi_0^2}\right)^2} \approx \frac{2z}{k\varpi_0}$$

k being the wave number and ϖ_0 the beam waist of the gaussian beam.

In this equation we can see that with lower values of $k\varpi_0$ the asymptotic slope will be increased, so the paraxial solution will not be an adequate approximation.

After this, to compare the two previous techniques we define the following error equation over the radiation surface.

$$\varepsilon = \frac{\sum_i \left| \left|Huy(x,y,z)_i\right|^2 - \left|\Psi_{0,0}(x,y,z)_i\right|^2 \right|}{\sum_i \left|Huy(x,y,z)_i\right|^2}$$

where $\left|Huy(x,y,z)_i\right|$ is the radiation pattern over each point of the radiation surface obtained with the Huygens method and $\left|\Psi_{0,0}(x,y,z)_i\right|$ is the gaussian expansion .

Results

To determine if the paraxial condition is satisfied, we will calculate the theoretical error (ε) for different values of $k\varpi_0$. This one can be observed in figure 1. For instance, in the case of $k\varpi_0$=6, an error of 6.9% is obtained. Using this figure, and fixing the maximum error that we can get, the paraxial condition can be found.

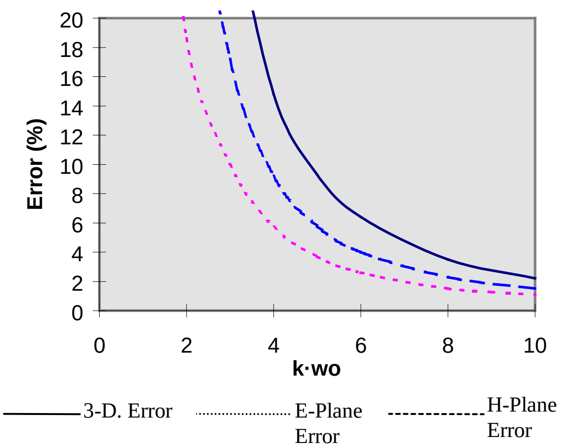

Figure 1 : 3-dimensional error (ε) for different $k\varpi_0$ values.

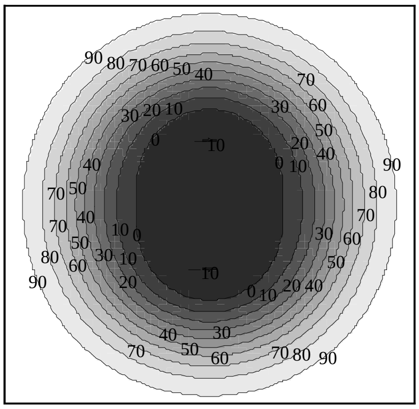

Figure 4: Surface error (%) between the Huygens and Gaussian propagation methods (figures 3 and 4 respectively) in the case of $k\varpi_0$=4.

In particular, we show some results for $k\varpi_0$=4. In figure 2 and 3, we can see the Huygens and Gaussian propagation radiation pattern respectively. Furthermore, in figure 4, we analyse the error surface between the two methods. The total error is 14.8%, so we can consider that the paraxial condition is not satisfied.

Conclusions

A comparison between the Huygens and Gaussian propagation methods to obtain the radiation pattern has been proposed and analysed.

Graphical with different error values in function of the $k\varpi_0$ values has been obtained. In this way, paraxial condition can be extracted for this graphic.

References

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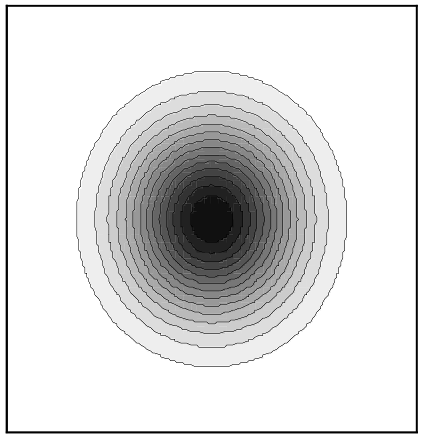

Figure 2: Radiation pattern obtained with Gaussian propagation with $k\varpi_0$=4.

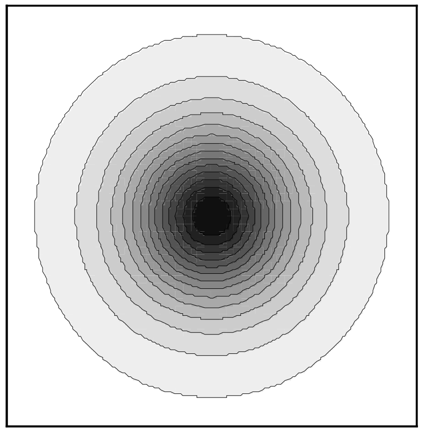

Figure 3: Radiation pattern obtained with Huygens method for $k\varpi_0$=4.

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